

Claim. Vertex Covers \leq_p Ind. Set
Ind. Set \leq_p Vertex Covers.

Claim. In any graph, a set $S \subseteq V$ is independent
iff \bar{S} is a vertex cover.

Pf. If S is independent, then no edge ~~is contained~~ ^{has both}
its endpoints in S . \Leftrightarrow any edge has at least
one endpoint in \bar{S} . $\Leftrightarrow \bar{S}$ is a vertex cover.

Pf (reduction) A graph has a vertex cover of size $\leq k$
 \Leftrightarrow the same graph has an ind. set of size $\geq n-k$.

The reduction: (G, k) \longrightarrow $(G, n-k)$
vertex cover ind. set problem

Proof of correctness: If G has a vertex cover ^{S} of
size $\leq k$, then by claim, \bar{S} is an ind. set.

$$|S| + |\bar{S}| = n, \quad |S| \leq k \Rightarrow |\bar{S}| \geq n-k$$

Hence, if (G, k) as a vertex cover problem has
answer yes, then $(G, n-k)$ as an ind set problem
has answer yes.

If G has no ~~ind. set~~ vertex cover of size $\leq k$,
then, suppose $(G, n-k)$ as an ind. set problem has
answer yes, that implies a vertex cover of size $\leq k$.