Matroids and the Greedy Algorithm

- Recall: On a universe E, we are given weight function $w : E \to \mathbb{R}_+$ and independent sets $\mathcal{M} \subseteq 2^E$. The weight of a set $S \subseteq E$ is $\sum_{a_i \in S} w(a_i)$. We want to output a maximal independent set with minimum weight.
- We assume $\mathcal{M}$ to be downward closed.
- The Greedy algorithm: sort the elements in E so that $w(a_1) \leq w(a_2) \leq \ldots \leq w(a_n)$; start with $S \leftarrow \emptyset$; then for each $i = 1, \ldots, n$, if $S \cup \{a_i\} \in \mathcal{M}$, $S \leftarrow S \cup \{a_i\}$.

Theorem

$\mathcal{M}$ is a matroid if and only if the greedy algorithm returns a maximal independent set with minimum weight for all weight functions.