Problem Set 1

Due date: Jan 26, 2017 at 9pm

1. (2 pts) Show by an example that the set of matchings in a graph does not necessarily constitute a matroid.

2. (5pt) Suppose an edge e is in a minimum spanning tree T of a grap G. If the weight of e decreases by Δ (while all other edges keep their weights unchanged), since T is still a spanning tree, the total weight of a minimum spanning tree obviously decreases by at least Δ. Could the decrease be more than Δ? If so, please give an example. If no, please give a proof.

3. (5pt) Given a graph G and any spanning tree T of it, denote by $\delta(T)$ the maximum degree of any node in T. (Recall that the degree of a node in T is the number of edges in T that are incident to the node.) Show that the problem of determining the minimum $\delta(T)$ over all spanning trees T of G is NP-hard.

4. (7pt) In class we gave an algorithm that finds a matching of maximum cardlnality matching in a bipartite graph in time $O(m\sqrt{n})$. We introduced the problem of maximum *weight* matching: given an undirected graph and nonnegative weights on its edges, find a matching that maximizes the total weight of edges in it. Give an algorithm that finds a maximum weight matching for a bipartite graph, which runs in polynomial time. (Recall that m is the number of edges, and n the number of nodes. Feel free to use any lemma or partial result we proved in class.)