$\forall$ universe $\mathcal{U}$. $2^{\mathcal{U}} := \{S : S \subseteq \mathcal{U}\}$.

$|\mathcal{U}| = n$, $n$ is even.

Every two sets of size $\frac{n}{2}$ are not comparable.

an Anti chain : $\{$sets of all subsets of size $\frac{n}{2}\}$.

This is the largest anti chain in $2^{\mathcal{U}}$.

(Sperner's Thm)

$\Rightarrow$ Theres's a chain covering using $\binom{n}{n/2}$ disjoint chains.

Ex. $\mathcal{U} = \{1, 2, 3\}$.

antichain: $\{1,2\} \nsubseteq \{2,3\}$. chain: $\emptyset \subseteq \{1\} \subseteq \{1,2\} \subseteq \{1,2,3\}$.

$\{2,3\} \nsubseteq \{1,2\}$.

Pf. (Dilworth's).

Given poset $S$, construct a bipartite graph $G = (U, V, E)$.

For each $s \in S$, put $u_s$ into $U$,

$v_s$ into $V$.

If $s \prec s'$, add edge $(u_s, v_{s'})$

Let $M$ be a max. matching of size $k$.

By the mid term problem 4, $\exists$ vertex cover $C$ of size $k$.

~~Look at nodes that are not in the vertex cover.~~

Constructing a chain covering:

~~If~~ Start with the trivial covering, where each chain is a singleton set of one element, and we need $n$ chains.

For each edge $(u_s, v_{s'})$ in $M$, merge the chain ending in $s$ and the one starting with $s'$, this reduces the number of chains by 1. So in the end we have $n-k$ chains that are disjoint and cover the set $S$.

Constructing an antichain $T$ of size $\geq n-k$:

Recall $C$ is a vertex cover of size $k$.

Pick an element $s$ to $T$ if $u_s \notin C$, and $v_s \notin C$.

So $|T| \geq n-k$. We claim $T$ is an antichain:

$\forall\ s, t \in T$, if $s \prec t$, then $(u_s, v_t) \in E$.

But by definition of $T$, $u_s \notin C$, $v_t \notin C$, so $(u_s, v_t)$ is not covered by $C$, contradiction.

So far we've found ~~a~~ $n-k$ disjoint covering chains and an antichain of size $\geq n-k$. So $T$ must be of size $n-k$. because the size of an antichain cannot be larger than # covering chains. $\blacksquare$