Pf. <u>Claim</u>. In any graph $G=(V,E)$,

$S \subseteq V$ is an independent set $\Leftrightarrow$ $V-S$ is a vertex cover.

Pf. If S is an independent set, take any edge $e=(u,v) \in E$. ~~It cann~~ u and v cannot both be in S, hence $u \in V \backslash S$, or $v \in V \backslash S$

$\Rightarrow V \backslash S$ is a ~~co~~ vertex cover.

If S is a vertex cover, take any two nodes $u, v \in V \backslash S$, then $(u,v) \notin E$. (o.w. contradicting S being a cover).

$\Rightarrow V \backslash S$ is an independent set.

<u>Reduction</u>: Input: $G=(V,E)$, k.

Output: $G=(V,E)$, $n-k$.

$n-k = |V|-k$

Pf: If Input is Ind. Set,

then by the claim, G has an ind set. S of size k

$\Leftrightarrow$ $V \backslash S$ (of size $n-k$) is a ~~co~~vertex cover.

Question: $A \le_p B \Rightarrow B \le_p A$?

Answer: No.

Claim. If A is NP-complete, and if A has a poly-time alg., then $NP = P$.

Pf. $\because \forall B \in NP$, $B \le_p A$,

$\Rightarrow B$ has a poly-time alg.

Highly unlikely that $NP = P$.

Pf. (Indep. Set is NP-Complete).

Step 1: Ind. Set $\in NP$.

Step 2. We show $3\text{-SAT} \le_p$ Ind. Set.

Given 3-SAT formula, construct for each clause three nodes, corresponding to the 3 literals.

Connect the three of them.

For every pair of nodes corresponding to a variable and its negate, draw an edge. Runs in poly time.

set $k = m \leftarrow$ # clauses.

3-SAT formula can be satisfied $\Leftrightarrow$ The Ind. Set instance has ind. set of size $k = m$.

$\Rightarrow$: Given value assignments that satisfy the formula, each clause contains ≥ 1 true literal, pick the corresponding node to the ind. set. Obviously m nodes are picked. Any two nodes picked are not in the same triangle (since they are from different clauses), and do not correspond to a variable and its negate (because we have a valid assignment). So they share no edge. Hence the set is an independent set.

$\Leftarrow$: $\exists$ Ind. Set of size $\geq m$ $\Rightarrow$ 3-SAT is satisfied.

Since each triangle can have ≤ 1 node in the independent set and there are m triangles, exactly one node from each triangle is in the independent set. For each node in the ind. set, if it corresponds to a variable, set that variable to TRUE, o.w. it corresponds to a variable's negate, set that variable to FALSE. Since ~~there's~~ no variable and its negate can have both their corresponding nodes in an ind. set, this is a valid assignment. Each clause is satisfied by this assignment since each contains a literal that's set to TRUE.