Reduction: Dom Set $\rightarrow$ k-Center

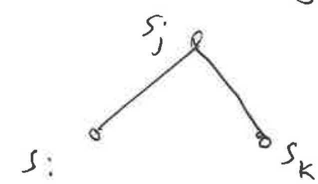
Given $G = (V, E)$, let these be n sites, $s_1, \dots, s_n$.

$V = \{v_1, \dots, v_n\}$.

If $(v_i, v_j) \in E$, then $d(s_i, s_j) = 1$

$(v_i, v_j) \notin E$, $d(s_i, s_j) = 2$

(Triangle ineq: $\forall i, j, k$, $d(s_i, s_j) + d(s_j, s_k) \geq d(s_i, s_k)$.



$\exists\ C \subseteq \{s_1, \dots, s_n\}$, $|C| = k$, with covering radius 1.

$S_1 = \{1, 2\}$

$S_2 = \{2, 3\}$

$S_3 = \{3, 4\}$

If optimal covering is $S_1^*, \dots, S_m^*$ with $S_i^* \cap S_j^* = \phi$.

then $\frac{OPT}{|U|}$ is the average per-~~cost~~ item cost of $S_1^*, \dots, S_m^*$.

In general, $S_i^* \cap S_j^* \neq \phi$, but if so, the actual per-item cost in opt covering is ~~higher~~ smaller than $\frac{OPT}{|U|}$.

$$\sum_{i \in C^*} \frac{w_i}{|S_i|} \cdot \frac{|S_i|}{\sum_{j \in C^*} |S_j|} \geq 0.$$

$$\sum_{i \in C^*} \frac{|S_i|}{\sum_{j \in C^*} |S_j|} = 1.$$